



## Exercise VIII, Theory of Computation 2025

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. Solve as many problems as you can and ask for help if you get stuck for too long. Problems marked \* are more difficult but also more fun :).

These problems are taken from various sources at EPFL and on the Internet, too numerous to cite individually.

1 Prove that the following problems are in **NP**:

1a A Boolean formula is said to be in *disjunctive normal form (DNF)* if it is an OR ( $\vee$ ) of a number of terms, where each term is an AND ( $\wedge$ ) of some literals. For instance, the following is a DNF formula:  $(x \wedge y \wedge \bar{z}) \vee (\bar{y} \wedge z) \vee (\bar{x} \wedge \bar{y})$ .

Given a DNF formula, decide it is satisfiable.

1b Given  $n$  positive integers  $a_1, a_2, \dots, a_n$ , decide if there is some  $S \subseteq \{1, 2, \dots, n\}$  with

$$\sum_{i \in S} a_i = \sum_{i \notin S} a_i.$$

2 Show that if  $L_1, L_2 \subseteq \Sigma^*$  are in **NP** then their concatenation  $L = L_1 L_2$  is also in **NP**.

3 Prove that the following problem is **NP**-complete: Given an undirected graph  $G = (V, E)$  and a positive integer  $k$ , decide if there is a subset  $S$  of  $V$  with  $|S| \geq k$  such that there is an edge between every pair of vertices in  $S$ ?

4\* Let 3SAT3 be the problem of deciding whether a 3CNF formula  $\varphi$ , with the additional assumption that every variable occurs at most 3 times in  $\varphi$ , is satisfiable. Show that 3SAT3 is **NP**-complete.